ON THE ESSENCE OF PHYSICAL RELATIVITY

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In an important investigation recently reported,[1] Mr. Leigh Page has determined an expression for the translatory force required to sustain an assigned varying velocity in an electrostatic system of a certain type, that namely which it has been usual to investigate as a model of an electron in analytical discussions. The expression contains terms involving the acceleration of the system and its time-gradients, but no term involving its velocity except in combination with these other quantities. The application of his formula to which Mr. Page restricts himself is to challenge a result that an isolated radiating system is subject to retarding force from the reaction of its own radiation of amount equal to its velocity multiplied by rate of radiation divided by c^2. The necessity for such refutation is based on the idea that on principles of relativity the velocity of an isolated body could have no meaning. But the formula obtained seems to leave this question as it was: for equally the acceleration of the isolated body could have no meaning; and moreover, though the thrust of the radiation is, in this case compensated, the velocity appears to be actually involved in the formula in the same manner as the impugned result would require for this particular problem of a convected electrostatic system, for which the radiation is extremely transient and very slight in the absence of extraneous force.

It has always seemed to me that this subject, which may be described as that of interaction of the æther with uniform motion, though of slight account phenomenally, is theoretically of high significance, in that it is destined perhaps to throw light on the nature of the forward momentum that is convected by radiation, and thence on the intimate dynamical nature of radiation itself and the physical function of the æther. I therefore propose briefly to indicate what I hold, provisionally of course, to be their present trend of knowledge. An adequate survey of this widely ramifying domain, in its various aspects, is difficult to compress, even when concentration and avoidance of detail is the main aim, and is therefore reserved for a communication to the *Physical Review*.

In the first place the plan now customary of regarding an electron as merely an aggregation of differential elements of electricity seems to be far too narrow a foundation for its internal properties, except by way of partial and provisional illustration. If electricity is to be explained as constituted of electrons, it can hardly be a step towards finality to explain electrons as constituted of electricity. On such a view the kinetic reaction of the inertia of an electron presents itself in theory under the artificial guise of a self-retarding force exerted

by the electron on itself, as given by Mr. Page's formula: moreover on this order of ideas there is no reason assignable why electricity should be aggregated into electrons at all, unless other forces and constraints extraneous to the electric theory are to be introduced to hold them together.

On the other hand an æther theory demands immediately the existence of electrons as the singularities in its own constitution—like vortex-rings in fluid —which determine its activity: and the electrons had already formed an essential part of electrodynamic theory before they were detected in the free state, and the actual magnitude of their inertia thereby ascertained in comparison with the masses of the atoms of the dynamical elements. As being thus self-subsisting mobile structures, whose interactions with each other at distances great compared with the dimensions of their nuclei are fully known, but whose internal constitution and adjacent influence could only be illustrated and guessed at, their dynamical specification will be most suitably, at any rate most hopefully, expressed in terms of their electric charge, their effective inertia, and their electrodynamic fields of known forms except inside and close up to the nuclei; all else being held in suspense as matter only for provisional speculation. As in the case of the simplest exemplar of a theory of a medium, the hydrodynamic theory for an ideal perfect fluid containing vortex rings or moving solids, this inertia may be aeolotropic: it may even when waves can travel be a function of the velocity instead of being constant. But it is to be the same whether the system which is its seat be moving with acceleration or not. The momentum of the system, of type mv, is not to involve its acceleration: nor is the energy of translatory motion of the body, of type $\frac{1}{2}m^1v^2$ where m^1 may be different from m. Otherwise all existing general dynamical theory would be dissolved. The production of acceleration is ascribed to applied force, which measured by its result as the value of $d(mv)/dt$. The formula for the force required to sustain an assigned varying motion of Mr. Page's special model of an electron does not in this way introduce only v and its acceleration f: the terms in it which involve the gradients of f have no place as yet in this scheme. Some origin extraneous to the electron must be assigned to them. On an æther-theory that cause is the radiation shot out from the electron while it is in varying motion, which convects somehow mechanical momentum away from it into the free state; the backward kick of this momentum acting on the source from which it is ejected is describable as the back-pressure of the radiation, which would of course exert a compensating forward kick on any obstacle, however distant, which absorbs it. However we may clothe our thought in language of relativity, it would appear that this issuing radiation does effectively possess an absolute velocity c, and therefore an absolute velocity of its source also is theoretically determinable from observations made in relation to it: and this seems to explain how it is that Mr. Page's formula for the force on a system entirely isolated can involve the acceleration of this system and its velocity also.

But a complete isolated system travelling with velocity v, when we include in the system all the radiation that has issued to however great a distance from its nucleus, cannot create new translational momentum. Therefore the backward pressure proportional to v of the radiational momentum projected from it namely $-v/c^2.dE/dt$ on this theory, must be compensated by disappearance of forward momentum attached to the source itself. The momentum of the source is mv: its rate of change is $mdv/dt+vdm/dt$; the first term is the usual kinetic reaction of the inertia m it has at the instant; the other term proportional to v is the compensating one here adumbrated. It follows that we must admit change of mass specified by the formulae $\delta m = \delta E/c^2$; so that the mass of a radiating body diminishes proportionally to the energy it radiates, except in so far as there is compensation provided by extraneous radiation which it absorbs.

For example, a particle of cosmic dust describing an orbit round the Sun is retarded by its own radiation, but its mass is kept up by the radiation it gains from the Sun: thus there is no compensation to the resisting force in this case, and the particle will gradually be sucked into the Sun, as Poynting in an important memoir showed.

The physical theory of electrodynamics, including optics, would thus, on the so-called classical lines—which can be held to be not yet obsolete, though special problems of expansion or development to adapt them to new experimental discovery are pressing —involve electrons specified in terms of charge and field and inertia, which are the link with matter, and it would involve also free radiation existing in the æther, specified as regards mechanical force by its convected secondary longitudinal momentum, but physically in evidence mainly by the type and energy of its transverse undulations. These are the materials out of which the science is to evolve itself: and the problem is whether that evolution can be gradually effected in a natural and consistent manner, and one which runs parallel with the course of evolution of experimental knowledge.

The true essence of the relativity of external knowledge is that we can investigate a system only in relation to some other system, and the most convenient perhaps the only feasible other system has been hitherto the ideal Newtonian frame of reference of space and time; for that is the canonical system, so to speak, with regard to which dynamical principles take on an ideal simple form, and it is a system which is being determined with continually expanding precision by the progress of astronomical science.

The special question now in evidence is—Is it now expedient to exchange this frame of reference, corresponding to c infinite, for another far more complex but very slightly different continuum having a finite space-time modulus c ? The more fundamental question is—Are we to assign to either frame dynamical properties, typified by propagation of physical effect in space in terms of undulations sustained by stress and inertia, or are we to assign to it properties solely geometrical and regard all physical effect as merely projected in duration across space? The forms of special unrestricted relativity which

have been recently current ultimately demand and perhaps prefer the latter course; and such modes of expression can apparently be elaborated so as to include most of optics, though perhaps in an artificial and unfruitful manner, if we replace the Newtonian scalar corpuscles of light by projected vector elements of fields of potential (not however conserved in value) from which forces would be analytically derivable. Mr. Page's work has improved very remarkably such a scheme of projected influence by showing that, provided c-relativity is postulated for space and time, it is elements of longitudinal electric force that may be regarded as projected from the sources, and are moreover conserved after the manner of *quanta* as they travel onward.

There is no absolute criterion to decide between the two ideals. The first order of ideas has proved itself as the foundation on which the interlaced fabric of electric and optical science has been actually constructed: the other seems to offer as yet only somewhat ingenuous and disjointed though significant expression for certain striking features of recent discovery which the former has not yet succeeded in assimilating, and seems to require us to obliterate the course of evolution of the science or perhaps to retain it as a mere historical survival.

All these modes of restatement of departments of physical science in more expressly relative terms may be comprehended as partial analytic developments of the far wider principle of the purely relational character of our external knowledge, which was advanced and systematically fortified with great abstract force in the general metaphysical domain by Bishop Berkeley; a principle by means of which he passed on to examine the criterion of real objective existence, and one which was well understood in its present aspect by his friends in Yale College nearly two centuries ago.

Thus in these matters we are hardly concerned with refuting any theory, for all are relative: it is fruitless to traverse any proposition, unless we take into account the definitions and context in which it subsists. The question is, as to which scheme of formulation gives as a whole the closest and most expressive representation of the complex of natural knowledge, and affords the most promising clue to its future elaboration and extension. But a choice does not by any means preclude development along other promising but provisional lines for which an interpretation has yet to be found. A main merit of Mr. Page's powerful papers, especially that of 1914, the feature which may be said to constitute them into a theory, is to my mind that they translate the usual so-called classical electrodynamic theory into the order of ideas, formulation under relativity of the c type being an essential feature, that describes physical action in terms of projected entities (which might perhaps even be *quanta*) of various kinds, effecting this translation in more intuitive and fundamental terms than had previously been attained to; and also that by their conciseness and geometrical directness they facilitate that comparison and contrast with the alternative order of ideas which is the essential matter.

[1] These PROCEEDINGS, **4**, 1918, (47–46); *Physic. Rev., Ithaca*, (Ser. 2), **11**, 1918, (376–400).